Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2013

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.
Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. (a) A ball is thrown vertically upwards with a speed of $44{\cdot}1\ \text{m s}^{-1}$.

Calculate the time interval between the instants that the ball is $39{\cdot}2$ m above the point of projection.

$$s = ut + \tfrac{1}{2}at^2$$

$$39{\cdot}2 = 44{\cdot}1t + \tfrac{1}{2}(-9{\cdot}8)t^2 \qquad 5$$

$$t^2 - 9t + 8 = 0$$

$$(t-1)(t-8) = 0$$

$$\Rightarrow \quad t = 1, \quad t = 8 \qquad 5,5$$

$$t_1 = 8 - 1$$

$$= 7\text{ s} \qquad 5$$

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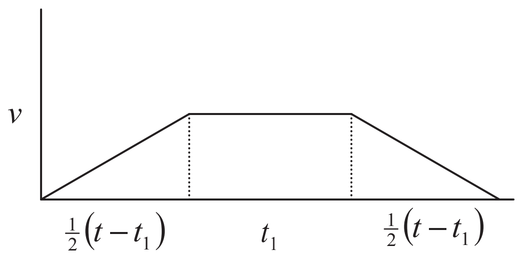
1. **(b)** A lift ascends from rest with constant acceleration *f* until it reaches a speed *v*. It continues at this speed for t_1 seconds and then decelerates uniformly to rest with deceleration *f*.
The total distance ascended is *d*, and the total time taken is *t* seconds.

(i) Draw a speed-time graph for the motion of the lift.

(ii) Show that $v = \frac{1}{2} f(t - t_1)$.

(iii) Show that $t_1 = \sqrt{t^2 - \frac{4d}{f}}$.

(i)



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(ii) $f = \dfrac{v}{\frac{1}{2}(t - t_1)} \Rightarrow v = \frac{1}{2} f(t - t_1)$ 5

(iii) $d = \frac{1}{4}(t - t_1)v + t_1 v + \frac{1}{4}(t - t_1)v$ or $\{d = \frac{1}{2}(t + t_1)v\}$ 5

$= \left(\frac{1}{2}t - \frac{1}{2}t_1 + t_1\right) v$

$d = \frac{1}{2}(t + t_1)\frac{1}{2} f(t - t_1)$ 5

$\frac{4d}{f} = t^2 - t_1^2$

$\Rightarrow \quad t_1 = \sqrt{t^2 - \frac{4d}{f}}$ 5

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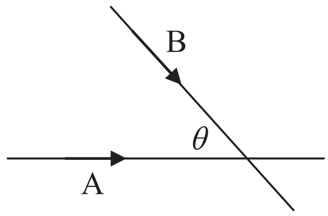
2. **(a)** Two cars, A and B, travel along two straight roads which intersect at an angle θ.

Car A is moving towards the intersection at a uniform speed of 9 m s^{-1}.

Car B is moving towards the intersection at a uniform speed of 15 m s^{-1}.

At a certain instant each car is 90 m from the intersection and approaching the intersection.



(i) Find the distance between the cars when B is at the intersection.

(ii) If the shortest distance between the cars is 36 m, find the value of θ.

(i) $|AB| = 90 - 9 \times \left(\frac{90}{15}\right)$ 5

$= 36 \text{ m}$ 5

(ii) $\vec{V}_A = 9\,\vec{i}$

$\vec{V}_B = 15\cos\theta\,\vec{i} - 15\sin\theta\,\vec{j}$ 5

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$= (9 - 15\cos\theta)\vec{i} + 15\sin\theta\,\vec{j}$ 5

$\vec{V}_{AB} \perp AB \Rightarrow 9 - 15\cos\theta = 0$

$\Rightarrow \theta = \cos^{-1}\left(\frac{9}{15}\right). = 53{\cdot}13°.$ 5

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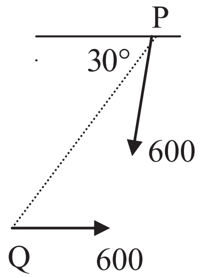
2 **(b)** An aircraft P, flying at 600 km h^{-1}, sets out to intercept a second aircraft Q, which is a distance away in a direction west 30° south, and flying due east at 600 km h^{-1}.



Find the direction in which P should fly in order to intercept Q.

$$\vec{V}_P = -600\cos\alpha\,\vec{i} - 600\sin\alpha\,\vec{j}$$ 5

$$\vec{V}_Q = 600\,\vec{i}$$ 5

$$\vec{V}_{PQ} = \vec{V}_P - \vec{V}_Q$$
$$= \{-600\cos\alpha - 600\}\vec{i} - 600\sin\alpha\,\vec{j}$$ 5

$$\tan 30 = \frac{600\sin\alpha}{600\cos\alpha + 600}$$ 5

$$\sqrt{3}\sin\alpha = \cos\alpha + 1$$
$$3\sin^2\alpha = \cos^2\alpha + 2\cos\alpha + 1$$
$$3\left(1-\cos^2\alpha\right) = \cos^2\alpha + 2\cos\alpha + 1$$
$$0 = 4\cos^2\alpha + 2\cos\alpha - 2$$
$$\cos\alpha = \frac{1}{2}$$
$$\Rightarrow \alpha = 60° \quad \Rightarrow \text{W } 60°\text{ S or S } 30°\text{ W}$$ 5 | 25

3. (a) A particle is projected from a point on horizontal ground.
The speed of projection is u m s^{-1} at an angle α to the horizontal.
The range of the particle is R and the maximum height reached by the particle is $\dfrac{R}{4\sqrt{3}}$.

(i) Show that $R = \dfrac{2u^2 \sin\alpha\cos\alpha}{g}$.

(ii) Find the value of α.

(i)

$$u\sin\alpha . t - \tfrac{1}{2}gt^2 = 0$$

$$t = \frac{2u\sin\alpha}{g} \qquad 5$$

$$R = u\cos\alpha . t$$

$$= u\cos\alpha \times \frac{2u\sin\alpha}{g}$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g} \qquad 5$$

(ii)

$$t_1 = \frac{u\sin\alpha}{g} \qquad 5$$

$$\frac{R}{4\sqrt{3}} = u\sin\alpha . t_1 - \tfrac{1}{2}gt_1^2$$

$$\frac{2u^2\sin\alpha\cos\alpha}{4g\sqrt{3}} = u\sin\alpha\left(\frac{u\sin\alpha}{g}\right) - \frac{g}{2}\left(\frac{u\sin\alpha}{g}\right)^2 \qquad 5$$

$$\frac{\cos\alpha}{2\sqrt{3}} = \sin\alpha - \tfrac{1}{2}\sin\alpha$$

$$\tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^\circ \qquad 5$$

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3 **(b)** A plane is inclined at an angle $\tan^{-1}\frac{1}{2}$ to the horizontal.

A particle is projected up the plane with initial speed u m s^{-1} at an angle θ to the inclined plane.

The plane of projection is vertical and contains the line of greatest slope.

Find the value of θ that will give a maximum range up the inclined plane.

$$r_j = 0$$ 5

$$u\sin\theta \times t - \tfrac{1}{2} g\cos\alpha \times t^2 = 0$$

$$t = \frac{2u\sin\theta}{g\cos\alpha} = \frac{u\sqrt{5}\sin\theta}{g}$$ 5

$$R = u\cos\theta \times t - \tfrac{1}{2} g\sin\alpha \times t^2$$

$$= \frac{u^2\sqrt{5}}{g}\left\{\cos\theta\sin\theta - \tfrac{1}{2}\sin^2\theta\right\}$$ 5

$$= \frac{u^2\sqrt{5}}{2g}\left\{\sin 2\theta - \sin^2\theta\right\}$$

$$\frac{dR}{d\theta} = \frac{u^2\sqrt{5}}{2g}\left\{2\cos 2\theta - 2\sin\theta\cos\theta\right\}$$ 5

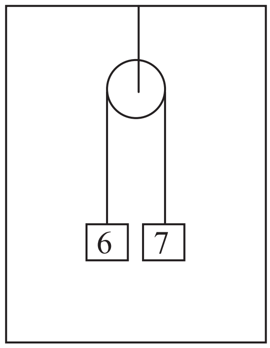
$$\frac{dR}{d\theta} = 0 \quad \Rightarrow 2\cos 2\theta - 2\sin\theta\cos\theta = 0$$

$$2\cos 2\theta = \sin 2\theta$$

$$\Rightarrow \quad \tan 2\theta = 2$$

$$\Rightarrow \quad \theta = 31{\cdot}7^\circ$$ 5 | 25

4. **(a)** Two particles of masses 6 kg and 7 kg are connected by a light inextensible string passing over a smooth light fixed pulley which is fixed to the ceiling of a lift.



The particles are released from rest.

Find the tension in the string

(i) when the lift remains at rest

(ii) when the lift is rising vertically with constant acceleration $\frac{g}{8}$.

(i)

$$7g - T = 7f \qquad 5$$

$$T - 6g = 6f \qquad 5$$

$$f = \frac{g}{13}$$

$$T = 6g + 6f$$

$$T = \frac{84g}{13} \quad \text{or} \quad 63{\cdot}32 \qquad 5$$

(ii)

$$\left.\begin{aligned} 7g - T &= 7\left(f - \frac{g}{8}\right) \\ T - 6g &= 6\left(f + \frac{g}{8}\right) \end{aligned}\right\} \qquad 5$$

$$f = \frac{9g}{104}$$

$$T = 6g + 6f + \frac{6g}{8}$$

$$T = \frac{189g}{26} \quad \text{or} \quad 71{\cdot}24 \qquad 5$$

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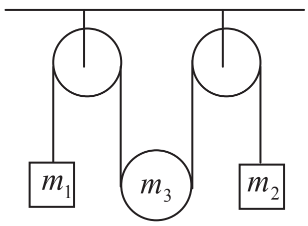
4 **(b)** A light inextensible string passes over a smooth fixed pulley, under a movable smooth pulley of mass m_3, and then over a second smooth fixed pulley.



A particle of mass m_1 is attached to one end of the string and a particle of mass m_2 is attached to the other end.

The system is released from rest.

Find the tension in the string in terms of m_1, m_2 and m_3.

$$T - m_1 g = m_1 p$$ 5

$$T - m_2 g = m_2 q$$ 5

$$m_3 g - 2T = m_3 \times \tfrac{1}{2}(p+q)$$ 5

$$m_3 g - 2T = \frac{m_3}{2}\left\{\frac{T}{m_1} - g + \frac{T}{m_2} - g\right\}$$ 5

$$2m_1 m_2 m_3 g - 4m_1 m_2 T = m_2 m_3 T + m_1 m_3 T - 2m_1 m_2 m_3 g$$

$$4m_1 m_2 m_3 g = T\{m_1 m_3 + m_2 m_3 + 4m_1 m_2\}$$

$$T = \frac{4m_1 m_2 m_3 g}{m_1 m_3 + m_2 m_3 + 4m_1 m_2}$$ 5

or $$\frac{4g}{\frac{1}{m_1} + \frac{1}{m_2} + \frac{4}{m_3}}$$

25

5. **(a)** A smooth sphere A, of mass $3m$, moving with speed u, collides directly with a smooth sphere B, of mass $5m$, which is at rest.

The coefficient of restitution for the collision is e. Find

(i) the speed, in terms of u and e, of each sphere after the collision

(ii) the value of e if the magnitude of the impulse imparted to each sphere as a result of the collision is $2mu$.

PCM $\quad 3m(u) + 5m(0) = 3mv_1 + 5mv_2$	5	
NEL $\quad v_1 - v_2 = -e(u-0)$	5	
$v_1 = \frac{u}{8}(3-5e)$, $\quad v_2 = \frac{3u}{8}(1+e)$	5	
$I = \lvert 5mv_2 - 0 \rvert$		
$2mu = \frac{15mu}{8}(1+e)$		
$16mu = 15mu(1+e)$		
$e = \frac{1}{15}$	5	20

5 **(b)** A ball is dropped on to a table and it rises after impact to one-quarter of the height of the fall.

(i) Find the value of the coefficient of restitution between the ball and the table.

If sheets of paper are placed on the table the coefficient of restitution decreases by a factor proportional to the thickness of the paper. When the thickness of the paper is 2·5 cm it rises to only one-ninth of the height of the fall.

(ii) Find the value of the coefficient of restitution between the ball and this thickness of paper.

(iii) What thickness of paper is required in order that the rebound will be one-sixteenth of the height of the fall?

(i)

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2gh$$

$$v = \sqrt{2gh}$$ 5

$$v^2 = u^2 + 2as$$

$$0 = \left(e\sqrt{2gh}\right)^2 - 2g\left(\frac{h}{4}\right)$$

$$\Rightarrow e = \frac{1}{2}$$ 5

(ii)

$$v^2 = u^2 + 2as$$

$$0 = \left(e_1\sqrt{2gh_1}\right)^2 - 2g\left(\frac{h_1}{9}\right)$$

$$\Rightarrow e_1 = \frac{1}{3}$$ 5

(iii)

$$v^2 = u^2 + 2as$$

$$0 = \left(e_2\sqrt{2gh_2}\right)^2 - 2g\left(\frac{h_2}{16}\right)$$

$$\Rightarrow e_2 = \frac{1}{4}$$ 5

$$e = k(2{\cdot}5)e_1 \Rightarrow \frac{1}{2} = k(2{\cdot}5)\times\frac{1}{3}$$

$$\Rightarrow k = 0{\cdot}6$$ 5

$$e = k(x)e_2 \Rightarrow \frac{1}{2} = 0{\cdot}6(x)\times\frac{1}{4}$$

$$\Rightarrow x = \frac{10}{3} \quad \text{or} \quad 3{\cdot}33\text{ cm.}$$ 5

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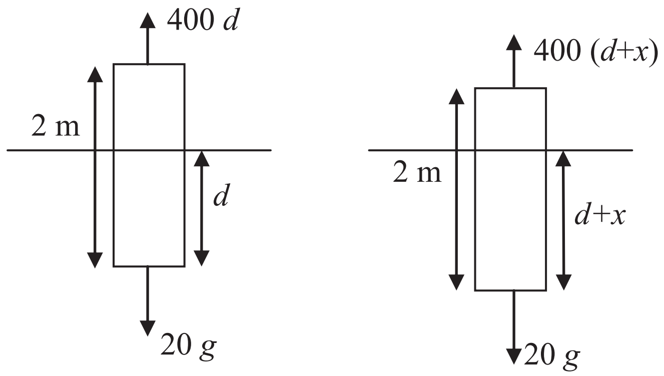
6. (a) A rectangular block of wood of mass 20 kg and height 2 m floats in a liquid. The block experiences an upward force of 400*d* N, where *d* is the depth, in metres, of the bottom of the block below the surface. Find

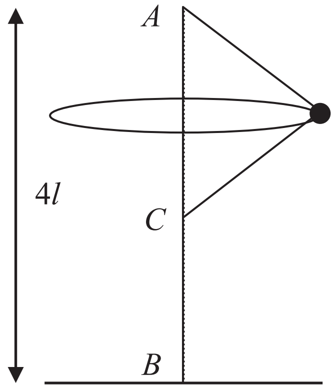
(i) value of *d* when the block is in equilibrium

(ii) the period of the motion of the block if it is pushed down 0·3 m from the equilibrium position and then released.



(*i*) $400d = 20g$		
$\Rightarrow d = 0{\cdot}49$	5	
(*ii*) $F = 20g - 400(d+x)$ $= -400x$	5	
$a = \dfrac{F}{m} = -20x$		
$\Rightarrow \omega = \sqrt{20}$ or $2\sqrt{5}$	5	
$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2\sqrt{5}}$ $= \dfrac{\pi}{\sqrt{5}}$ or 1·4 s.	5	20

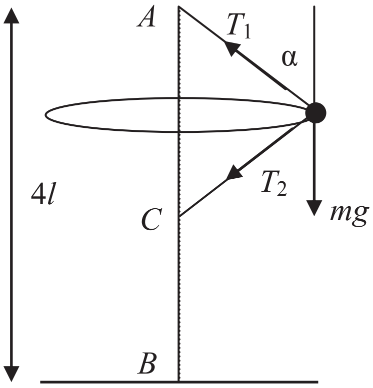
6 (b) A vertical rod BA, of length $4l$, has one end B fixed to a horizontal surface with the other end A vertically above B. The ends of a light inextensible string, of length $4l$, are fixed to A and to a point C, a distance $2l$ below A on the rod.



A small mass m kg is tied to the mid-point of the string. It rotates, with both parts of the string taut, in a horizontal circle with uniform angular velocity ω.

(i) Find the tension in each part of the string in terms of m, l and ω.

(ii) At a given instant both parts of the string are cut.
Find the time (in terms of l) which elapses before the mass strikes the horizontal surface.

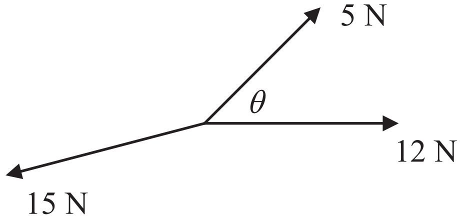


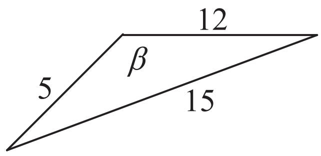
(i)	$\alpha = 60°$	5	
	$T_1 \cos 60 - T_2 \cos 60 = mg$	5	
	$T_1 - T_2 = 2mg$		
	$T_1 \sin 60 + T_2 \sin 60 = mr\omega^2$	5	
	$= m\left(\ell\sqrt{3}\right)\omega^2$		
	$T_1 + T_2 = 2m\ell\omega^2$		
	$T_1 = m\left(\ell\omega^2 + g\right)$, $T_2 = m\left(\ell\omega^2 - g\right)$	5	
(ii)	$s = ut + \frac{1}{2}at^2$		
	$3\ell = 0 + \frac{1}{2}gt^2$	5	
	$\Rightarrow t = \sqrt{\frac{6\ell}{g}}$	5	30

7. **(a)** Two forces 5 N and 12 N are inclined at an angle θ as shown in the diagram.

They are balanced by a force of 15 N.

Find the acute angle θ.





$$15^2 = 12^2 + 5^2 - 2 \times 12 \times 5 \times \cos\beta$$ 5, 5

$$225 = 144 + 25 - 120\cos\beta$$

$$\cos\beta = -\frac{56}{120} = -0{\cdot}4667$$

$$\beta = 117{\cdot}82°$$ 5

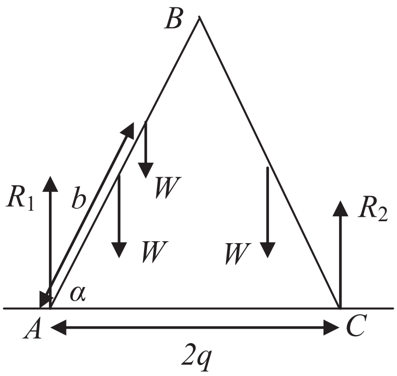
$$\Rightarrow \theta = 62{\cdot}18°$$ 5

20

7. **(b)** Two uniform rods AB and BC, of length 1 and weight W, are hinged at B and rest in equilibrium on a smooth horizontal plane.
A weight W is attached to AB at a distance b from A as shown in the diagram.
A light inextensible string AC of length $2q$ prevents the rods from slipping.

(i) Find the reaction at A and the reaction at C.

(ii) Show that the tension in the string is $\dfrac{q(1+b)W}{2\sqrt{1-q^2}}$.



(i)

$$R_2(2q) = W\left(\tfrac{1}{2}q\right) + W\left(\tfrac{3}{2}q\right) + W(b\cos\alpha)$$ 5

$$\cos\alpha = q$$

$$R_2(2q) = W\left(\tfrac{1}{2}q\right) + W\left(\tfrac{3}{2}q\right) + W(bq)$$

$$R_2 = \frac{W(2+b)}{2}$$ 5

$$R_1 + R_2 = 3W$$ 5

$$R_1 = \frac{W(4-b)}{2}$$ 5

(ii)

$$R_2(q) = W\left(\tfrac{1}{2}q\right) + T(\sin\alpha)$$ 5

$$\frac{W(2+b)q}{2} = \frac{Wq}{2} + T\left(\sqrt{1-q^2}\right)$$

$$\frac{W(1+b)q}{2} = T\left(\sqrt{1-q^2}\right)$$

$$T = \frac{W(1+b)q}{2\sqrt{1-q^2}}$$ 5

30

8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

$$\text{Let } \quad \text{M} = \text{mass per unit area}$$

$$\text{mass of element} = \text{M}\{2\pi\, x\, \text{d}x\}$$

$$\text{moment of inertia of the element} = \text{M}\{2\pi\, x\ \ \text{d}x\}x^2 \qquad 5$$

$$\text{moment of inertia of the disc} = 2\pi\,\text{M}\int_0^r x^3\,\text{d}x \qquad 5$$

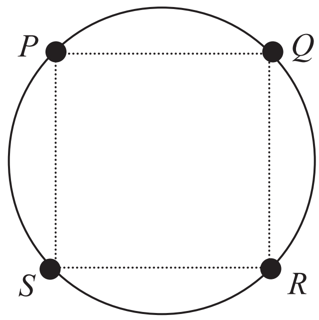
$$= 2\pi\,\text{M}\left[\frac{x^4}{4}\right]_0^r \qquad 5$$

$$= \text{M}\pi\,\frac{r^4}{2}$$

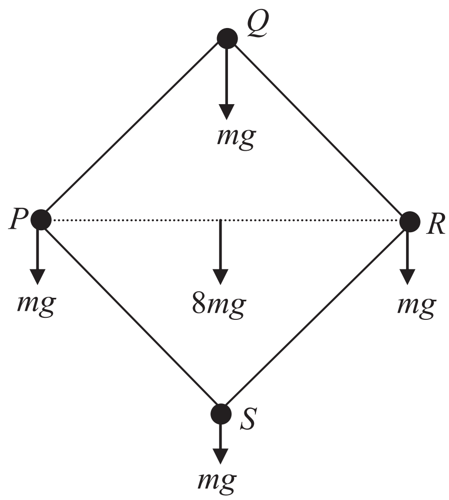
$$= \tfrac{1}{2}mr^2 \qquad 5 \qquad 20$$

8. **(b)** A uniform circular lamina, of mass $8m$ and radius r, can turn freely about a horizontal axis through P perpendicular to the plane of the lamina.

Particles each of mass m are fixed at four points which are on the circumference of the lamina and which are the vertices of square $PQRS$.
The compound body is set in motion.



Find **(i)** the period of small oscillations of the compound pendulum
(ii) the length of the equivalent simple pendulum.



(i)

$$Mgh = 8mg(r) + mg(r) + mg(r) + mg(2r)$$
$$= 12mgr$$

5,5

$$I = \left\{\tfrac{1}{2}(8m)r^2 + (8m)r^2\right\} + m\left(r\sqrt{2}\right)^2 + m\left(r\sqrt{2}\right)^2 + m(2r)^2$$
$$= 20mr^2$$

5,5

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$
$$= 2\pi\sqrt{\frac{20mr^2}{12mgr}} = 2\pi\sqrt{\frac{5r}{3g}}$$

5

(ii)

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5r}{3g}}$$
$$L = \frac{5r}{3}$$

5

30

9. **(a)** V_1 cm^3 of liquid A of relative density 0·8 is mixed with V_2 cm^3 of liquid B of relative density 0·9 to form a mixture of relative density 0·88.

The mass of the mixture is 0·44 kg.

Find the value of V_1 and the value of V_2.

$m_A + m_B = m_M$		
$800 \times V_1 + 900 \times V_2 = 880(V_1 + V_2)$	5	
$20V_2 = 80V_1$		
$V_2 = 4V_1$	5	
$880(V_1 + V_2) = 0{\cdot}44$	5	
$880(V_1 + 4V_1) = 0{\cdot}44$		
$V_1 = 0{\cdot}0001\ \text{m}^3$ or $100\ \text{cm}^3$	5	
$V_2 = 0{\cdot}0004\ \text{m}^3$ or $400\ \text{cm}^3$	5	25

9 **(b)** Liquid C of relative density 0·8 rests on liquid D of relative density 1·2 without mixing. A solid object of density ρ floats with part of its volume in liquid D and the remainder in liquid C.

The fraction of the volume of the object immersed in liquid D is $\dfrac{\rho - 2a}{a}$.

Find the value of a.

$W = B_C + B_D$	5	
$\rho(V_1 + V_2)g = 800V_1 g + 1200V_2 g$	5, 5	
$(\rho - 800)V_1 = (1200 - \rho)V_2$		
$\dfrac{V_2}{V_1 + V_2} = \dfrac{1}{\dfrac{V_1}{V_2} + 1}$		
$= \dfrac{1}{\dfrac{1200 - \rho}{\rho - 800} + 1}$	5	
$= \dfrac{\rho - 800}{1200 - \rho + \rho - 800}$		
$= \dfrac{\rho - 800}{400}$		
$\Rightarrow a = 400$	5	25

10. **(a)** If

$$x^2 \frac{dy}{dx} - 7 = 0$$

and $y = 1$ when $x = 7$, find the value of y when $x = 14$.

(b) A particle starts from rest at O at time $t = 0$. It travels along a straight line with acceleration $(24t - 16)$ m s^{-2}, where t is the time measured from the instant when the particle is at O. Find

(i) its velocity and its distance from O at time $t = 3$

(ii) the value of t when the speed of the particle is 80 m s^{-1}.

(c) Water flows from a tank at a rate proportional to the volume of water remaining in the tank. The tank is initially full and after one hour it is half full.

After how many more minutes will it be one-fifth full?

(a)

$$x^2 \frac{dy}{dx} = 7$$

$$\int dy = 7\int x^{-2}\, dx$$

$$\int_1^y dy = 7\int_7^{14} x^{-2}\, dx$$

$$[y]_1^y = 7\left[\frac{-1}{x}\right]_7^{14}$$

5

$$y - 1 = 7\left(\frac{-1}{14} + \frac{1}{7}\right)$$

$$y = 7\left(\frac{1}{14}\right) + 1$$

$$y = 1{\cdot}5$$

5

10

(*b*) (i)

$$\frac{dv}{dt} = 24t - 16$$

$$\int_0^v dv = \int_0^3 (24t - 16)\,dt$$

$$v = \left[12t^2 - 16t\right]_0^3$$

$$v = 60 \text{ m s}^{-1}$$ **5**

$$\int_0^s ds = \int_0^3 (12t^2 - 16\text{t})\,dt$$

$$s = \left[4t^3 - 8t^2\right]_0^3$$ **5**

$$s = 36 \text{ m}$$ **5**

(*b*) (ii)

$$v = 12t^2 - 16t$$

$$80 = 12t^2 - 16t$$

$$3t^2 - 4t - 20 = 0$$

$$(3t - 10)(t + 2) = 0$$

$$\Rightarrow t = \frac{10}{3} \text{ s}$$ **5** **20**

(*c*)

$$\frac{dV}{dt} = -kV$$ **5**

$$\int_{\text{V}}^{\frac{1}{2}V} \frac{1}{V}\,dV = -k\int_0^1 dt$$

$$\left[\ln\frac{V}{2} - \ln V\right] = -k$$

$$\Rightarrow k = \ln 2 \quad \text{or} \quad 0{\cdot}693$$ **5**

$$\left[\ln\frac{V}{5} - \ln V\right] = -kt$$

$$t = \frac{\ln 5}{\ln 2} = 2{\cdot}322 \text{ h}$$ **5**

$$t_1 = t - 1 = 79{\cdot}3 \text{ min}$$ **5** **20**

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